Vector

(i) Level Surface:

- level surfaces generated by $f(x,y,z)$ are $f(x,y,z) = c$ [where c is arbitary constant]

(ii) Level Curves:

- If (x,y) be a scalar function of two variable x,y $f(x,y)=c$. then level curves generated by [where c is arbitary constant].

(iii) Position vector of point in 3D $= (x\hat{i} + y\hat{j} + z\hat{k})$

$$\vec{r} = \text{P.V of } (x,y,z) = x\hat{i} + y\hat{j} + z\hat{k}$$

$\Rightarrow \dfrac{d\vec{r}}{dt}$ = velocity vector | $\dfrac{d^2 r}{dt^2}$ = Acceleration vector

* Formula:

i) $\dfrac{d}{dt}[\vec{c}] = 0$, where $\vec{c}$ is constant vector

ii) $\dfrac{d}{dt}[k\vec{F}(t)] = \dfrac{k\,df(t)}{dt}$, k is scalar [constant]

iii) $\dfrac{d}{dt}[\vec{u}(t) \pm \vec{v}(t)] = \dfrac{d\vec{u}}{dt} \pm \dfrac{d\vec{v}}{dt}$, where, $\vec{v}$, $\vec{u}$ are vector function.

iv) $\dfrac{d}{dt}[\vec{u}\cdot\vec{v}] = \vec{u}\cdot\dfrac{dv}{dt} + \dfrac{d\vec{u}}{dt}\cdot\vec{v}$

v) $\dfrac{d}{dt}[\vec{u}\times\vec{v}] = \vec{u}\times\dfrac{dv}{dt} + \dfrac{d\vec{u}}{dt}\times\vec{v}$

(vii) $\frac{d}{dt}[\vec{u}\cdot\vec{v}\cdot\vec{w}] = \left[\frac{d\vec{u}}{dt}, \vec{v}, \vec{w}\right] + \left[u, \frac{dv}{dt}, \vec{w}\right]$

$+ \left[\vec{u}, \vec{v}, \frac{d\vec{w}}{dt}\right]$

(vii) $\frac{d}{dt}\{\vec{u}\times(\vec{v}\times\vec{w})\} = \frac{d\vec{u}}{dt}\times(\vec{v}\times\vec{w}) +$

$\vec{u}\times\left(\frac{d\vec{v}}{dt}\times\vec{w}\right) + \vec{u}\left(\vec{v}\times\frac{dw}{dt}\right)$

(viii) $\frac{d}{dt}[\phi(t)\cdot\vec{u}(t)] = \phi(t)\frac{d\vec{u}}{dt} + \vec{u}(t)\frac{d\phi}{dt}$

Note

If $\vec{r} = \vec{r}(t)$ be the eq. of some curve then $\frac{dr}{dt}$ represent the direction of tangent vector.

* Gradient

$$\text{grad}(F) = \vec{\nabla}F = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)F$$

$$= \hat{i}\frac{\partial F}{\partial x} + \hat{j}\frac{\partial F}{\partial y} + \hat{k}\frac{\partial F}{\partial z}$$

→ It calculated for sclar function and after calculation it became vector function.

→ grad f is the rate of change of f.

→ [Geometrical] : grad F represent a normal vector to level curve, $f(x,y) = C$

* Directional Derivative:

Let $F(x,y,z)$ be a sclar function then directional derivative of $F(x,y,z)$ in the direction of $\vec{b}$ is given as $(\vec{\nabla}F).\hat{b}$ where $\hat{b}$ is unit vector along $\vec{\nabla}$ $\vec{b}$.

→ Directional Derivative gives us rate of change of F in the direction of b.

→ Maximum rate of change of any sclar valued function $F(x,y,z)$ is $|\vec{\nabla}F|$ and it occur in the direction of $\vec{\nabla}F$.

→ maximum rate of change of any scalar function $F(x,y,z)$ is $|\vec{\nabla}F|$ and it occurs in the direction of ~~b~~ $-(\nabla F)$ or opposite to $\vec{\nabla}F$.

* Divergence:

Let $\vec{v}(x,y,z)$ be a vector function, then divergence of $\vec{v}$ (x,y,z) $= div(\vec{v}) = \underbrace{\frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}}_{\text{diverge of } \vec{v} \text{ as sclar function.}}$

$\vec{v} = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$

- $\underbrace{\vec{\nabla}.\vec{v}}_{\text{divergence}} \neq \underbrace{\vec{v}.\vec{\nabla}}_{\text{operator}}$

- <u>Solenoidal</u> <u>vector</u>:

  If div $\vec{v} = 0$ then $\vec{v}$ is solenoidal vector

* Curl

Let $\vec{v} = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$ be a vector function.

$$\text{Curl } \vec{v} = \vec{\nabla} \times \vec{v} = \hat{i} \times \frac{\partial v}{\partial x} + \hat{j} \times \frac{\partial \vec{v}}{\partial y} + \hat{k} \times \frac{\partial v}{\partial z}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$\hat{i}\left[\frac{\partial v_3}{\partial y} - \frac{\partial v_2}{\partial z}\right] - \hat{j}\left[\frac{\partial v_3}{\partial x} - \frac{\partial v_1}{\partial z}\right] + \hat{k}\left[\frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y}\right]$$

Curl $\vec{v}$ is a vector function.

Irrotational vector:

If Curl $\vec{v} = 0$ $\Rightarrow \vec{v}$ is irrotational vector.

Formula

(i) Curl(grad ϕ) $= 0$, where ϕ is a scalar function.

(ii) div[Curl $\vec{v}$] $= 0$, where $\vec{v}$ is a vector function

(iii) div $[\phi \vec{u}]$ = grad $\phi \cdot \vec{u} + \phi$ div $\vec{u}$, where ϕ is scalar function and $\vec{u}$ is vector function

$$\Rightarrow \vec{\nabla}\cdot[\phi\vec{u}] = \nabla\phi\cdot\vec{u} + \phi(\vec{\nabla}\cdot\vec{u})$$

(iv) Curl $[\phi\vec{u}]$ = grad $\phi \times \vec{u} + \phi$ Curl $\vec{u}$

$$\Rightarrow \vec{\nabla} \times [\phi\vec{u}] = \vec{\nabla}\phi \times \vec{u} + \phi[\vec{\nabla} \times \vec{u}]$$

(v) $\operatorname{div}[\vec{u} \times \vec{v}] = \vec{v} \cdot \operatorname{curl}\vec{u} - \vec{u} \cdot \operatorname{curl}\vec{v}$

$= [\vec{u} \times \vec{v}] \cdot \nabla = \vec{\nabla} \cdot \{\vec{\nabla} \times \vec{u}\} - \vec{u} \cdot (\vec{\nabla} \times \vec{v}]$

(vi) $\operatorname{curl}(\vec{u} \times \vec{v}) = (\vec{v} \cdot \vec{\nabla})\vec{u} - (\vec{u} \cdot \vec{\nabla})\vec{v}$

$+ \vec{u}(\vec{\nabla} \cdot \vec{v}) - \vec{v}(\vec{\nabla} \cdot \vec{u})$

(vii) $\operatorname{grad}(\vec{u} \cdot \vec{v}) = (\vec{v} \cdot \vec{\nabla}) + (\vec{u} \cdot \vec{\nabla})\vec{v} +$

$\vec{v} \times \operatorname{curl}\vec{u} + \vec{u} + \operatorname{curl}\vec{v}$

(viii) $\operatorname{curl}(\operatorname{curl}\vec{v}) = \operatorname{grad}(\operatorname{div}\vec{v}) - \nabla^2\vec{v}$

$\vec{\nabla}(\vec{\nabla} \times \vec{v}) = \vec{\nabla}\{\vec{\nabla} \cdot \vec{v}] - \nabla V^2$, where

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, Laplacian operator.